EEE 391 Recitation 6

1)

Define $x(t)$ as

$$x(t) = 5\sqrt{2}\cos(20\pi t + \pi/4) + A\cos(20\pi t + \phi) \quad (1)$$

where A is a *positive* number. In addition, assume that $x(t)$ has a phase of zero, so that it may be written as

$$x(t) = B\cos(20\pi t), \quad (2)$$

where B is a *positive* number.

(a) What relationship must exist between A and ϕ in order for $x(t)$ to have zero phase as indicated in Eq. 2?

(b) If $B = 10$, what are the values for A and ϕ?

$$x(t) = 5\sqrt{2}\cos\left(20\pi t + \frac{\pi}{4}\right) + A\cos(2\pi t + \phi)$$

$$= B\cos(20\pi t)$$

$A > 0$

$B > 0$

Let $\underline{X}$ be the phasor representing $x(t)$

$$\underline{X} = 5\sqrt{2}\, e^{j\pi/4} + A e^{j\phi} = B$$

$$= 5\sqrt{2}\cos\frac{\pi}{4} + j5\sqrt{2}\sin\frac{\pi}{4} + A\cos\phi + jA\sin\phi = B$$

$$= \left(5\sqrt{2}\,\frac{\sqrt{2}}{2} + A\cos\phi\right) + j\left(5\sqrt{2}\,\frac{\sqrt{2}}{2} + A\sin\phi\right) = B$$

a) Thus: for zero phase: $\begin{cases} \text{Imaginary part zero} \\ \text{Real part positive} \end{cases}$

$$\boxed{\begin{array}{l} 5 + A\sin\phi = 0 \\ 5 + A\cos\phi > 0 \end{array}}$$

b) For $B=10$, solve: $\begin{cases} 5 + A\sin\phi = 0 \\ 5 + A\cos\phi = 10 \end{cases} \Rightarrow \begin{array}{l} A\sin\phi = -5 \\ A\cos\phi = 5 \end{array}$

$$\Rightarrow \underbrace{A^2\sin^2\phi + A^2\cos^2\phi}_{= A^2} = (-5)^2 + (5)^2 = 25 + 25 = 50$$

$$\Rightarrow \boxed{A = \sqrt{50}}$$

and $\left.\begin{array}{l} \cos\phi = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}} \\ \sin\phi = -\frac{5}{\sqrt{50}} = -\frac{1}{\sqrt{2}} \end{array}\right\} \rightarrow \boxed{\phi = -\pi/4}$

2)

1. (35 pts.) Consider a LTI system described by

$$y(n) = x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2),$$

where x(n) and y(n) denote input and output sequences, respectively.

(a) Find and plot impulse response h(n) of this system.

(b) Find the frequency response $H(e^{jw})$ of the system, and plot it for $-\pi \le w \le \pi$ rad.

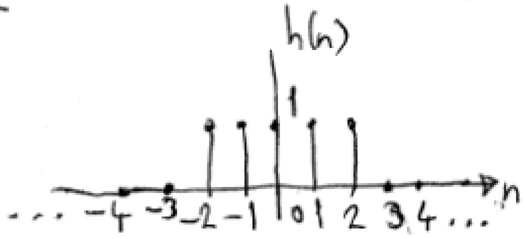
(c) Is this system stable or not? Is it causal or not?

(d) Find a recursive difference equation expressing this system.

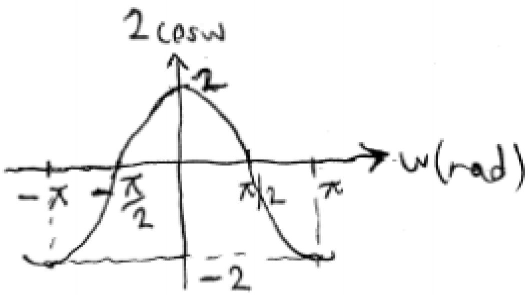
(e) Find a difference equation for the "inverse system" of this system.

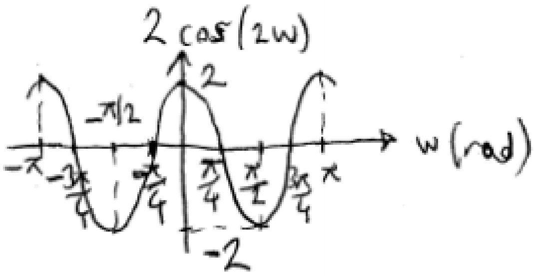
Answer: (a) $h(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2)$

$$= \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$



(b) $H(e^{jw}) = e^{-j2w} + e^{-jw} + 1 + e^{jw} + e^{j2w} = 1 + 2\cos w + 2\cos(2w)$,



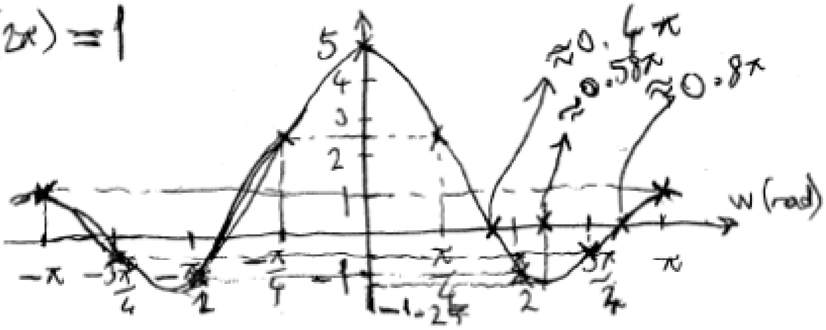


$H(e^{j0}) = 5$, $H(e^{j\pi/4}) = 1 + 2\underbrace{\cos(\pi/4)}_{1/\sqrt{2}} + 2\underbrace{\cos(\pi/2)}_{0} = 1 + \sqrt{2} \approx 2.4142$

$H(e^{j\pi/2}) = 1 + 2\cos(\pi/2) + 2\cos(\pi) = -1$

$H(e^{j3\pi/4}) = 1 + 2\cos(3\pi/4) + 2\cos(3\pi/2) = 1 - \sqrt{2} \approx -0.4142$

$H(e^{j\pi}) = 1 + 2\cos(\pi) + 2\cos(2\pi) = 1$



(c) $\sum_n |h(n)| = 5 < \infty \Rightarrow$ stable system.

$h(-1) \neq 0,\ h(-2) \neq 0 \Rightarrow$ <u>non</u>causal system.

(d) $y(n) = x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2)$

$y(n+1) = x(n+3) + x(n+2) + x(n+1) + x(n) + x(n-1)$

$\therefore\ y(n+1) - y(n) = x(n+3) - x(n-2) \Rightarrow \boxed{y(n) - y(n-1) = x(n+2) - x(n-3)}$

(e) $y(n) \longrightarrow \boxed{h_{inv}(n)} \longrightarrow x(n)$

$x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2) = y(n)$

$$\boxed{x(n) + x(n-1) + x(n-2) + x(n-3) + x(n-4) = y(n-2)}$$

or $x(n+2) - x(n-3) = y(n) - y(n-1) \Rightarrow \boxed{x(n) - x(n-5) = y(n-2) - y(n-3)}$

$x(n)$: output, $y(n)$: input of the inverse system.

Both of them are correct.

3)

A signal $x(t)$ is periodic with period $T_0 = 8$. Therefore it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/8)kt}.$$

It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{8}\int_{-4}^{0} (4+t)e^{-j(2\pi/8)kt}\,dt. \quad (1)$$

(a) In the expression for a_k in Equation (1) above, the integral and its limits define the signal $x(t)$. Determine an equation for $x(t)$ that is valid over one period.

(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-10 \le t \le 10$ seconds. Label it carefully.

(c) Determine a_0, the DC value of $x(t)$.

$T_0 = 8$ (sec) $\quad x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{j\left(\frac{2\pi}{8}\right)kt} \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{8}$

Fourier coefficients: $\quad a_k = \frac{1}{8}\int_{-4}^{0}(4+t)\,e^{-j\frac{2\pi}{8}kt}\,dt$

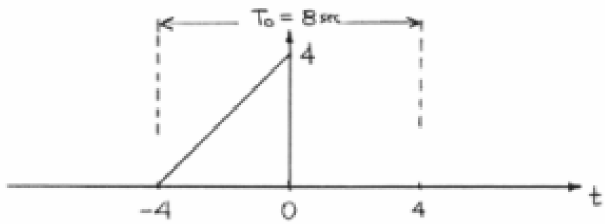
(a) The Fourier coefficients are given in general by:

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\,e^{-j\left(\frac{2\pi}{T_0}k\right)t}\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,e^{-j\frac{2\pi}{T_0}kt}\,dt$$

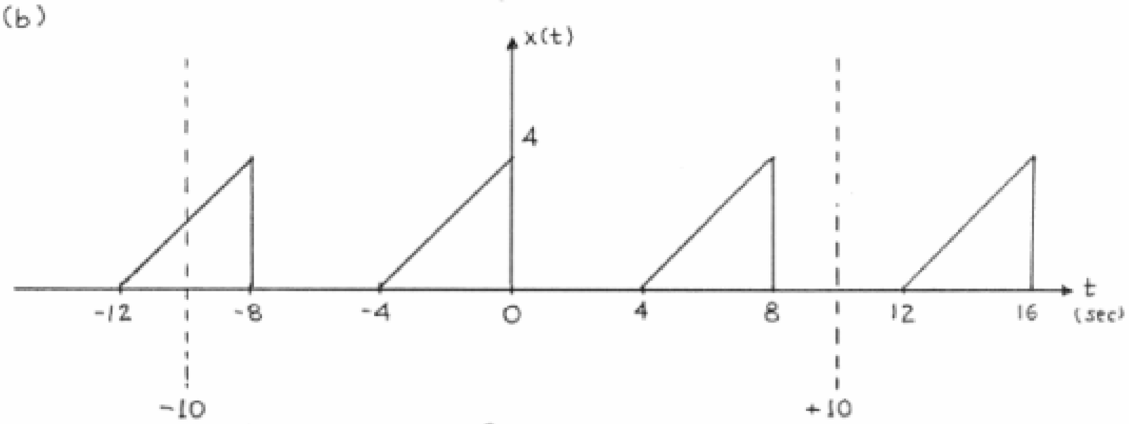
(any interval of length T_0)

From the given a_k it can be observed that: $\quad (T_0 = 8)$

$$x(t) = \begin{cases} 4+t & -4 \le t \le 0 \\ 0 & 0 \le t \le 4 \end{cases} \quad t \text{ in sec}$$



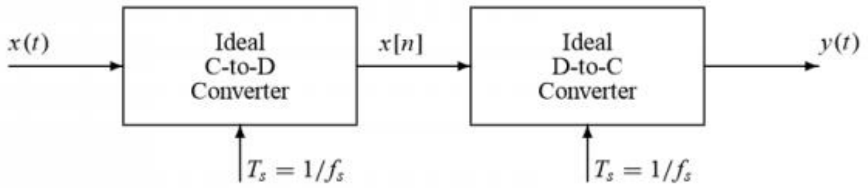
(b)



(c) $a_0 = \frac{1}{8}\int_{-4}^{4} x(t)\,dt = \frac{1}{8}\int_{-4}^{0}(t+4)\,dt = \frac{1}{8}\left(\frac{t^2}{2}+4t\right)\Big|_{-4}^{0} = 1$

4)

Consider the following system.



Suppose that a discrete-time signal $x[n]$ is given by the formula

$$x[n] = 4\cos(0.125\pi n + \pi/8)$$

If the sampling rate of the C-to-D converter is $f_s = 2000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 2000 Hz; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/2000$ secs.

Since $x[n]$ is given in sinusoids form, we suggest $x_1(t) = A_1\cos(2\pi f_1 t + \varphi_1)$ in which the parameters A_1, f_1, and φ_1 have to be found.

we use $x[n] = x_1(nT_s)$ where $T_s = \frac{1}{2000}$

$\Rightarrow 4\cos(0.125\pi n + \frac{\pi}{8}) = A_1\cos(2\pi f_1 \frac{n}{2000} + \varphi_1)$

$\Rightarrow A_1 = 4$, $\varphi_1 = \frac{\pi}{8}$, $f_1 = 125$ HZ

Therefore, $x_1(t) = 4\cos(2\pi(125)t + \frac{\pi}{8})$

To find $x_2(t)$ (that would give the same $x[n]$), we use the fact that adding $2\pi k$ ($k = \pm 1, \pm 2, \ldots$) to $\hat{\omega}$ (the frequency of $x[n]$) does not change anything. i.e., $x[n] = 4\cos((0.125\pi + 2k\pi)n + \frac{\pi}{8})$

If we start with $x_2(t) = A_2\cos(2\pi f_2 t + \varphi_2)$, then $x_2(t)$ can be found from $x_2(nT_s) = x[n]$

Following the same approach we had for $x_1(t)$,

we obtain:

$$A_2 = 4 \quad , \quad \varphi_2 = \frac{\pi}{8} \quad ,$$

$$\frac{2\pi f_2}{2000} = 0.125\pi + 2K\pi \qquad K = \pm 1, \pm 2, \cdots$$

$$\rightarrow f_2 = 125 + 2000K \qquad K = \pm 1, \pm 2, \cdots$$

Since we require $f_2 < 2000$, thus we choose

$K = -1 \rightarrow f_2 = -1875$

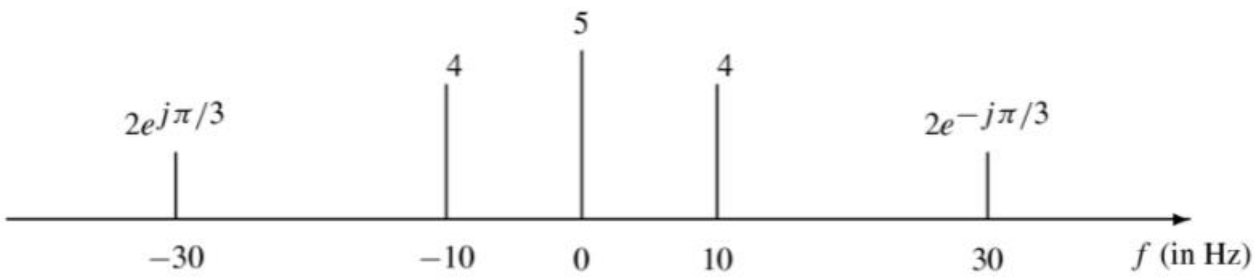
$$\rightarrow x_2(t) = 4\cos(-2\pi(1875)t + \frac{\pi}{8})$$
$$= 4\cos(2\pi(1875)t - \frac{\pi}{8})$$

because $\cos(-\theta) = \cos(\theta)$ for any θ

5)

PROBLEM:

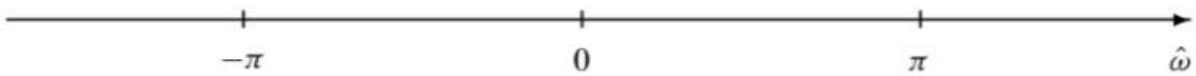
A signal $x(t)$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x(t)$.

(b) Is the signal $x(t)$ periodic? If so, what is the period?

(c) The signal $x(t)$ is sampled with sampling frequency $f_s = 1/T_s = 50$ samples/second to obtain the discrete-time signal $x[n] = x(nT_s)$. Write an equation for $x[n]$ and plot the spectrum of $x[n]$ for normalized frequencies $-\pi \leq \hat{\omega} \leq \pi$.



(a) $x(t) = 2e^{j\pi/3}e^{-j2\pi(30)t} + \underbrace{4e^{-j2\pi(10)t} + 4e^{j2\pi(10)t}}_{8\cos(2\pi(10)t)} + 5$

$+ 2e^{-j\pi/3}e^{j2\pi(30)t}$

$x(t) = 4\cos(60\pi t - \pi/3) + 8\cos(20\pi t) + 5$

(b) $\cos(60\pi t) \longrightarrow$ period $= 2\pi/60\pi = 1/30$ sec

$\cos(20\pi t) \longrightarrow$ period $= 2\pi/20\pi = 1/10$ sec.

Since $\frac{1}{30}$ divides $\frac{1}{10}$, the period of $x(t)$ is $\frac{1}{10}$ sec

OR 30 Hz is three times 10 Hz, so the fundamental frequency is 10 Hz $\Rightarrow$ period $= 1/10$ sec.

(c) $F_s = 1/T_s = 50$

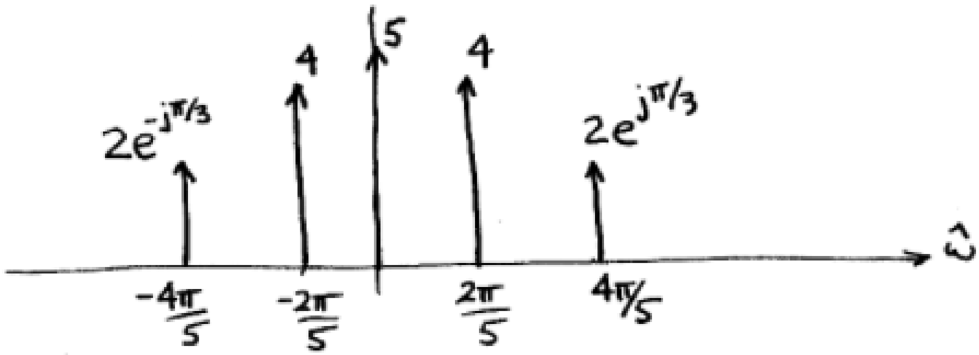
$x[n] = x(nT_s) = x(n/F_s)$

$= 4\cos(60\pi n/50 - \pi/3) + 8\cos(20\pi n/50) + 5$

$= 4\cos(\frac{6\pi n}{5} - \pi/3) + 8\cos(\frac{2\pi}{5}n) + 5$ $\quad \frac{6\pi}{5} = 2\pi - \frac{4\pi}{5}$

$= 4\cos(2\pi n - \frac{4\pi}{5}n - \pi/3) + 8\cos(\frac{2\pi}{5}n) + 5$

$= 4\cos(\frac{4\pi}{5}n + \pi/3) + 8\cos(\frac{2\pi}{5}n) + 5$



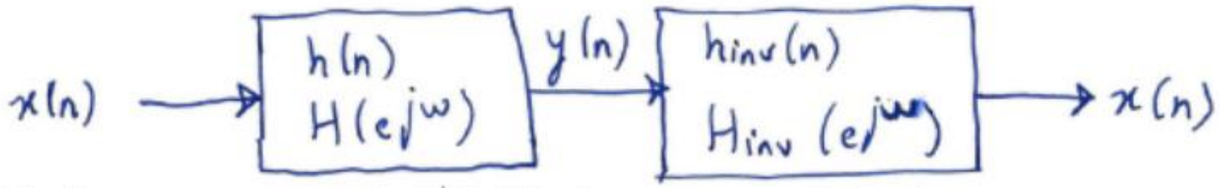
6)

2. (35 pts.) Consider a LTI system described by the following difference equation:

$$y(n) - (3/4)\, y(n-1) + (1/8)\, y(n-2) = 2\, x(n-1)$$

with $y(-1) = y(-2) = 0$. $x(n)$ and $y(n)$ denote input and output sequences, respectively.

(a) Find the frequency response $H(e^{jw})$ of this system. (10 pts.)
(b) Find the impulse response $h(n)$ of this system. (Hint: Apply partial fraction expansion before taking the inverse DTFT.) Is this system causal? Is it stable? (10 pts.)
(c) Consider the inverse system of this system:



Find the frequency response $H_{inv}(e^{jw})$ of the inverse system. (7.5 pts.)
(d) Find the impulse response $h_{inv}(n)$ of the inverse system. Is the inverse system causal? Is it stable? (7.5 pts.)

2. (a) $y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = 2\, x(n-1)$

Let's take Fourier tr. of both sides: $Y(e^{jw}) = \text{DTFT}\{y(n)\}$,
$X(e^{jw}) = \text{DTFT}\{x(n)\}$

and use $\text{DTFT}\{y(n-k)\} = e^{-jwk} \cdot Y(e^{jw})$:

$$\left[1 - \frac{3}{4} e^{-jw} + \frac{1}{8} e^{-j2w}\right] Y(e^{jw}) = 2e^{-jw} X(e^{jw})$$

$$\boxed{H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{2e^{-jw}}{1 - \frac{3}{4} e^{-jw} + \frac{1}{8} e^{-j2w}}}$$

(b) $h(n) = \text{IDTFT}\{H(e^{j\omega})\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$

where $H(e^{j\omega}) = \frac{2e^{-j\omega}}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-j2\omega}} = \frac{2e^{-j\omega}}{(1-\frac{1}{4}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})}$

$$\frac{1}{(1-\frac{1}{4}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})} = \frac{2}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

(3)

$$H(e^{j\omega}) = \frac{4e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}} - \frac{2e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}$$

$$\boxed{h(n) = 4\cdot(1/2)^{n-1}u(n-1) - 2\cdot(1/4)^{n-1}u(n-1),}$$

with $u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

$h(n) = 0$ for $n < 0 \Rightarrow$ the system is <u>causal.</u>

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=1}^{\infty} \left| 4\cdot(1/2)^{n-1} - 2\cdot(1/4)^{n-1} \right|$$

$$= \sum_{n=0}^{\infty} \left[4\cdot(1/2)^n - 2\cdot(1/4)^n\right] = 4\cdot\sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n - 2\cdot\sum_{n=0}^{\infty}\left(\tfrac{1}{4}\right)^n$$

$$= 4\cdot\frac{1}{1-1/2} - 2\cdot\frac{1}{1-1/4} = 8 - 8/3 = 16/3 < \infty$$

$\therefore$ The system is <u>stable.</u>

(c) $H_{inv}(e^{j\omega}) = \dfrac{X(e^{j\omega})}{Y(e^{j\omega})} = \dfrac{1}{H(e^{j\omega})} = \dfrac{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}{2e^{-j\omega}}$

$$= \boxed{\frac{1}{2}e^{j\omega} - \frac{3}{8} + \frac{1}{16}e^{-j\omega}}$$

(d) $H_{inv}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_{inv}(n)e^{-j\omega n} \Rightarrow h_{inv}(n) = \frac{1}{2}\delta(n+1) - \frac{3}{8}\delta(n) + \frac{1}{16}\delta(n-1)$

$$\boxed{h_{inv}(n) = \begin{cases} 1/2, & n=-1 \\ -3/8, & n=0 \\ 1/16, & n=1 \\ 0, & \text{otherwise} \end{cases}}$$

$h_{inv}(-1) = 1/2 \neq 0$

$\Downarrow$

the system is noncausal.

---

(4)

$\sum_n |h(n)| = \frac{1}{2} + \frac{3}{8} + \frac{1}{16} = \frac{15}{16} < \infty \Rightarrow$ the system is stable.

7)

A linear time-invariant system is described by the FIR difference equation

$$y[n] = x[n] - 3x[n-1] + 9x[n-2] - 3x[n-3] + x[n-4]$$

(a) Write a simple formula for the magnitude of the frequency response $|H(e^{j\hat{\omega}})|$. Express your answer in terms of real-valued functions only.

$$H(e^{j\hat{\omega}}) = 1 - 3e^{-j\hat{\omega}} + 9e^{-j2\hat{\omega}} - 3e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$= e^{-j2\hat{\omega}}\left(e^{j2\hat{\omega}} - 3e^{j\hat{\omega}} + 9 - 3e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right)$$

$$= e^{-j2\hat{\omega}}\left(2\cos 2\hat{\omega} - 6\cos\hat{\omega} + 9\right)$$

$\Rightarrow$

$$|H(e^{j\hat{\omega}})| = 2\cos 2\hat{\omega} - 6\cos\hat{\omega} + 9$$

(b) Derive a simple formula for the phase of the frequency response $\angle H(e^{j\hat{\omega}})$.

$\angle H(e^{j\hat{\omega}}) = -2\hat{\omega}$ from part (a)

8)

Question.

Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period.

(a) $x_1(t) = je^{j10t}$ **(b)** $x_2(t) = e^{(-1+j)t}$ **(c)** $x_3[n] = e^{j7\pi n}$
(d) $x_4[n] = 3e^{j3\pi(n+1/2)/5}$ **(e)** $x_5[n] = 3e^{j3/5(n+1/2)}$

Solution.

(a) $x_1(t)$ is a periodic complex exponential.

$$x_1(t) = je^{j10t} = e^{j(10t+\frac{\pi}{2})}$$

The fundamental period of $x_1(t)$ is $\frac{2\pi}{10} = \frac{\pi}{5}$.

(b) $x_2(t)$ is a complex exponential multiplied by a decaying exponential. Therefore, $x_2(t)$ is not periodic.

(c) $x_3[n]$ is a periodic signal.

$$x_3[n] = e^{j7\pi n} = e^{j\pi n}$$

$x_3[n]$ is a complex exponential with a fundamental period of $\frac{2\pi}{\pi} = 2$.

(d) $x_4[n]$ is a periodic signal. The fundamental period is given by $N = m(\frac{2\pi}{3\pi/5}) = m(\frac{10}{3})$. By choosing $m = 3$, we obtain the fundamental period to be 10.

(e) $x_5[n]$ is not periodic. $x_5[n]$ is a complex exponential with $\omega_0 = 3/5$. We cannot find any integer m such that $m(\frac{2\pi}{\omega_0})$ is also an integer. Therefore, $x_5[n]$ is not periodic.

9)

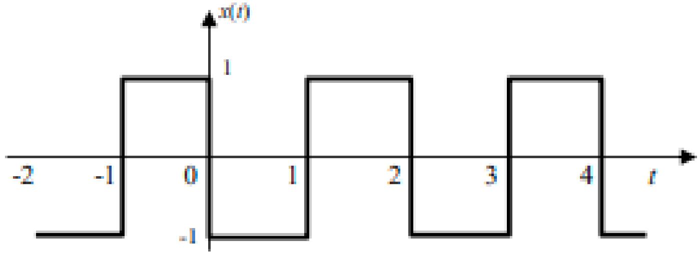
Q07. A periodic function $x(t)$ is given by the following equation .

$$x(t)=\begin{cases}-1 & 0<t<\\ +1 & 1<t<\end{cases}$$

a. Plot the function $x(t)$. (04)
b. Determine the fundamental frequency of the signal. (04)
c. Find D.C. component of the signal (a_0). (04)
d. Find the Fourier series components (a_k). (08)
e. Plot the first three frequency spectrum ($a_{\pm1}$, $a_{\pm2}$, $a_{\pm3}$). (05)

---

a.



b. $T_0 = 2$ s; $f_0 = 1/T_0 = 1/2 = 0.5$ Hz

f_0 = 0.5 Hz

c. $a_0=\frac{1}{T_0}\int_0^{T_0}x(t)dt=\frac{1}{2}\left\{\int_0^1(-1)dt+\int_1^2 dt\right\}=\frac{1}{2}\times\left\{-t\big|_0^1+t\big|_1^2\right\}=\frac{1}{2}\times(-1+0+2-1)=0$

$a_0 = 0$

d.

$$a_k=\frac{1}{T_0}\int_0^{T_0}x(t)e^{-j(2\pi/T_0)kt}dt=\frac{1}{2}\left\{\int_0^1(-1)e^{-j(2\pi/2)kt}dt+\int_1^2 e^{-j(2\pi/2)kt}dt\right\}$$

$$a_k=\frac{1}{2}\times\left\{\frac{-e^{-j(2\pi/2)kt}}{-j(2\pi/2)k}\bigg|_0^1+\frac{e^{-j(2\pi/2)kt}}{-j(2\pi/2)k}\bigg|_1^2\right\}=\frac{j}{2\pi k}\times\left(-e^{-j\pi k}+1+e^{-j2\pi k}-e^{-j\pi k}\right)$$

$$a_k=\frac{j(1+e^{-j2\pi k}-2e^{-j\pi k})}{2\pi k}=\frac{j(1+1-2e^{-j\pi k})}{2\pi k}=\frac{j(2-2(-1)^k)}{2\pi k}=\frac{j(1-(-1)^k)}{\pi k}$$

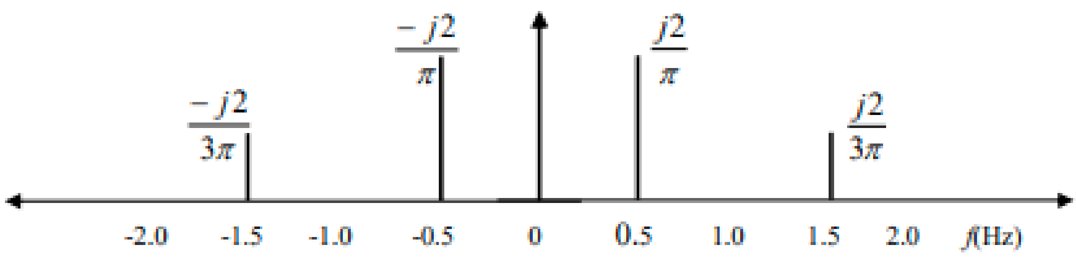
$$\boldsymbol{a_k=\frac{j(1-(-1)^k)}{\pi k}}$$

e.

for $k = 1$; $a_1=\frac{j(1-(-1)^1)}{\pi}=\frac{j(1+1)}{\pi}=\frac{j2}{\pi}$ for $k = -1$; $a_{-1}=\frac{j(1-(-1)^{-1})}{-\pi}=\frac{-j(1+1)}{\pi}=\frac{-j2}{\pi}$

for $k = 2$; $a_2=\frac{j(1-(-1)^2)}{\pi 2}=\frac{j(1-1)}{2\pi}=0$ for $k = -2$; $a_{-2}=\frac{j(1-(-1)^{-2})}{-\pi 2}=\frac{-j(1-1)}{2\pi}=0$

for $k = 3$; $a_3=\frac{j(1-(-1)^3)}{\pi 3}=\frac{j(1+1)}{3\pi}=\frac{j2}{3\pi}$ for $k = -3$; $a_{-3}=\frac{j(1-(-1)^{-3})}{-\pi 3}=\frac{-j(1+1)}{3\pi}=\frac{-j2}{3\pi}$



10)

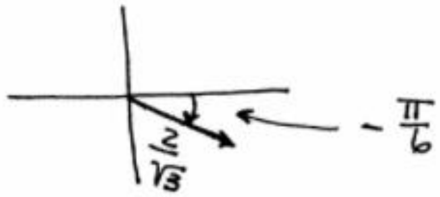
Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form. Let

$$V = -\frac{1}{\sqrt{3}} - j.$$

(a) Express jV in polar form. In addition plot jV as a vector.

$$jV = -j\frac{1}{\sqrt{3}} + 1$$

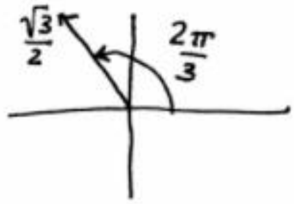
$$= \boxed{\frac{2}{\sqrt{3}} e^{-j\frac{\pi}{6}}}$$



(b) Express the inverse of V in rectangular form. In addition plot $\frac{1}{V}$ as a vector.

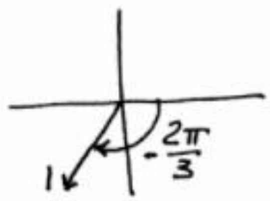
$$V = \frac{2}{\sqrt{3}} e^{-j\frac{2\pi}{3}}$$

$$\frac{1}{V} = \frac{\sqrt{3}}{2} e^{j\frac{2\pi}{3}} = \boxed{-\frac{\sqrt{3}}{4} + j\frac{3}{4}}$$



(c) If $Z = \frac{|V|}{V^*}$, express Z in polar form. In addition plot Z as a vector.

$$Z = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} e^{j\frac{2\pi}{3}}} = \boxed{e^{-j\frac{2\pi}{3}}}$$



(d) Express $\Re e\{j^3 V e^{j15t}\}$ in the standard "cosine" form.

$$\mathcal{R}e\left\{j^3 V e^{j15t}\right\} = \mathcal{R}e\left\{e^{-j\frac{\pi}{2}} \cdot \frac{2}{\sqrt{3}} e^{-j\frac{2\pi}{3}} e^{j15t}\right\} = \mathcal{R}e\left\{\frac{2}{\sqrt{3}} e^{j\frac{5\pi}{6}} e^{j15t}\right\}$$

$$\boxed{\begin{gathered} x(t) = \frac{2}{\sqrt{3}} \cos\left(15t + \frac{5\pi}{6}\right) \\ \text{or} \\ x(t) = \frac{2}{\sqrt{3}} \cos\left(15t - \frac{7\pi}{6}\right) \end{gathered}}$$